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Short Communication

Reducing spatial data using an optimized regressive discrete Fourier series

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Abstract

With the development of optical measurement techniques it is possible to obtain vast amounts of data. In vibrometry applications in particular operational deflection shapes are often obtained with very high spatial resolution. Fortunately, many techniques exist to reduce (approximate) the measurement data. One of the most common techniques for evaluating optical measurement data is by means of a Fourier analysis. However, this technique suffers from what is known as leakage when a non-integer number of periods is considered. This gives rise to non-negligible errors, which will obviously hamper the accuracy of the synthesized shape. Another technique such as a discrete cosine transform, used in the widely spread -jpeg standard does not suffer these anomalies but can still prove erroneous at times. One of the more recent approaches is via a so-called regressive discrete Fourier series (introduced by Arruda) which suffers one disadvantage. The problem statement is nonlinear in the parameters and needs *a priori* information about the deflection shape. This can be resolved by using the optimized regressive discrete fourier series (ORDFS), introduced in this article, which uses a nonlinear least squares approach. In this article the method will be applied in particular to the reduction of data for laser vibrometer measurements performed on an inorganic phosphate cement (IPC) beam (1D), as well as on a car door (2D). The proposed technique will also be validated on simulations to illustrate the properties concerning compression ratio and synthesized mode shape error. The introduced method will be bench marked for compression ratio and synthesized deflection shape error with all prior mentioned techniques.

1. Introduction

There are many ways to achieve data reduction. An important and frequently used approach is to apply a Fourier decomposition where the signal (e.g. the operational deflection shapes (ODS)) is approximated by means of a series of DFT lines [1]. This approximation, however, introduces a distortion effect called leakage. This is due to the assumption that the signal at hand is periodic within the measurement window. A common solution to this problem is using windows to reduce the effect, which however broadens the frequency resolution. A more accurate technique that also offers powerful compression capabilities is the discrete cosine transform (DCT) [2]. This technique is the basis of the widely spread -jpeg standard for compressing images.

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More recently a powerful technique was introduced by J.R.F. Arruda [3,4] using regressive discrete Fourier series (RDFS). The technique suffers one disadvantage, namely the fact that the basic problem statement is nonlinear in the parameters. This was solved initially by choosing a random period for the desired signal, which in a lot of cases is not entirely accurate. In Refs. [5–7] this problem was circumvented by fitting a general pole-residue model on the data (referred to as GRDFS), thereby estimating all unknown parameters (frequency, phase, damping and residues). While proving quite robust and accurate, the latter approach does exhibit a quite large computational load. In this article an approach is shown, which is based again on the RDFS method, but one that is computationally much lighter than the GRDFS but more accurate than the traditional RDFS. This is done by estimating the *a priori* unknown period in a robust fashion using a nonlinear approach. In the following sections the method will be described and a comparison with the classical DFT, DCT and RDFS reduction techniques will be made. In Section 3 simulations will be shown illustrating the novel technique's reduction capacity and accuracy. In Section 4 experiments will be shown on an inorganic phosphate composite (IPC) beam and on a rear car door and finally some conclusions will be drawn in the last section.

2. Regressive discrete Fourier series

2.1. Background of the regressive Fourier technique

Regressive Fourier series were first introduced by Arruda [3,4]. The technique boils down to representing a periodic sequence by a series of sines and cosines, like a Fourier series, however, by tuning the period of the series to reduce the leakage effect. In depth detail on the method can be found in the previously mentioned articles as well as in Refs. [6,8].

For clarities sake the two-dimensional representation of a mode shape $z(n,m)$ with constant spatial resolution Δx and Δy by a regressive discrete Fourier series will be shown here (the one-dimensional equations are analogous). The length of the data in x is $M\Delta x$ and the length of the data in y is $N\Delta y$:

$$z(n,m)=\sum_{k_x=0}^{N-1}\sum_{k_y=0}^{M-1}S(k_x,k_y)\mathrm{e}^{\mathrm{i}2\pi k_x n/N}\mathrm{e}^{\mathrm{i}2\pi k_y m/M}. \tag{1}$$

with $n=0\ldots N$ and $m=0\ldots M$ and $S(k_x,k_y)$ represent the two-dimensional discrete Fourier coefficients:

$$S(k_x,k_y)=\frac{1}{N}\frac{1}{M}\sum_{n=0}^{N}\sum_{m=0}^{M}z(n,m)\mathrm{e}^{\mathrm{i}2\pi-k_x n/N}\mathrm{e}^{\mathrm{i}2\pi-k_y m/M}. \tag{2}$$

The method proposed by Arruda now stipulates that the data should also be represented by a two-dimensional discrete Fourier series, but where the periods of the Fourier series αN and βM are not equal to the original data lengths N and M in x- and y-direction, respectively:

$$z(n,m)=\sum_{k_x=-p}^{p}\sum_{k_y=-q}^{q}S(k_x,k_y)\mathrm{e}^{\mathrm{i}2\pi k_x n/\alpha N}\mathrm{e}^{\mathrm{i}2\pi k_y m/\beta M}. \tag{3}$$

The data reduction is achieved because the number of components p and q are far lower than the original data lengths N and M. The unknown coefficients $S(k_x,k_y)$ can be determined by formulating a least-squares problem. Define a matrix $Z_{\alpha N}(k_x,n)=\mathrm{e}^{\mathrm{i}2\pi k_x n/\alpha N}$ and $Z_{\beta M}(k_y,m)=\mathrm{e}^{\mathrm{i}2\pi k_y m/\beta M}$. This allows Eq. (3) to be written as

$$\mathbf{z}=\mathbf{Z_{\alpha N}SZ_{\beta M}}. \tag{4}$$

The matrix $\mathbf{S}$ containing the unknown coefficients can now be determined and its size is $(2p+1)\times(2q+1)$:

$$\mathbf{S}=(\mathbf{Z_{\alpha N}}^{\mathrm{t}}\mathbf{Z_{\alpha N}})^{-1}\mathbf{Z_{\alpha N}}^{\mathrm{t}}\mathbf{z}\mathbf{Z_{\beta M}}^{\mathrm{t}}(\mathbf{Z_{\beta M}Z_{\beta M}}^{\mathrm{t}})^{-1}, \tag{5}$$

where t denotes a complex conjugate transpose of a matrix.

Now in practice the factor α (and β in the two-dimensional case) is usually not known beforehand so it should be estimated as well together with the residues leading to a nonlinear least-squares problem. In the next section, an approach is introduced based on a regressive discrete Fourier series which allows estimation of these parameters.

2.2. Data reduction

To understand how the data reduction is achieved one can consider the following complex-valued multiharmonic function:

$$z(x) = \sum_{l=0}^{L} a_l \mathrm{e}^{-\sigma_l x + \mathrm{i}\omega_l x}. \tag{6}$$

Evaluating this function at discrete locations $x = n\Delta x, \Delta x$ denoting a constant spatial resolution and $n = 0 \ldots N-1$, results in the following sequence:

$$z[x] = z(n\Delta x) = \sum_{l=0}^{L} a_l \lambda_l^n \tag{7}$$

with $\lambda_l = \mathrm{e}^{-\sigma_l \Delta x + \mathrm{i}\omega_l \Delta x}$ and ω_l, σ_l denoting respectively the spatial frequency and damping of component l in the operational deflection shape.

The reduction of data is possible due to the fact that the number of components L with which one models the complete sequence in Eq. (6) is much smaller than the number of measurement points N. Thus data reduction for the ODS discussed further on in this article means applying the proposed algorithm to the raw shape itself.

2.3. Optimized regressive discrete Fourier series (ORDFS)

As was stated in the previous section the arbitrary factor α (and β for the two-dimensional case) is not known *a priori*. This means that Eq. (5) and its one-dimensional counterpart are nonlinear in the parameters. To resolve this, a non-linear least-squares approach can be used. Using the built-in function `lsqnonlin` of the signal processing toolbox [9] in Matlab, it is possible to estimate the unknown parameters α (and β) together with the unknown coefficients S. This function uses a classical Gauss–Newton iterative procedure to solve this problem. The starting values for α (and β) can be chosen arbitrarily, in an interval $[0 \ldots 2]$. Usually the initial values can be chosen equal to one (which makes the frequency lines coincide with the used DFT grid). However, when these initial values are far off from the final values, the Gauss–Newton algorithm should be replaced by the more robust Levenberg–Marquardt optimization. In the following only the latter algorithm will be withheld.

However, the parameters α (and β) are not the same for every vibration pattern $Z(\omega)$. Therefore, when calculating the reduced FRF-matrix $\mathbf{H}_{\text{virtual}}$ an optimized value for α (and β) should be found over the entire frequency range. This can be done by creating a global mode shape $\mathbf{Z}_{\text{global}}$ which contains all the vibration patterns in the frequency band of interest:

$$\mathbf{Z}_{\text{global}} = \sum_{\omega} \mathbf{Z}(\omega) \tag{8}$$

This global mode shape can then be transformed using Eq. (3), but where the parameters α and β are now global parameters.

$$\mathrm{Z}_{\text{global}}(x, y) = \sum_{k_x=-p}^{p} \sum_{k_y=-q}^{q} S(k, l) \mathrm{e}^{\mathrm{i}2\pi \frac{k_x n}{\alpha N}} \mathrm{e}^{\mathrm{i}2\pi \frac{k_y m}{\beta M}}. \tag{9}$$

These global parameters α and β can now be used in the calculation of $\mathbf{Z}_{\text{virtual}}$ at every frequency ω.

3. Simulations

In this section the ORDFS approach will be tested with respect to the number of necessary components to obtain a fit of the data. The technique is compared with the classical DCT which is used in, e.g. the popular jpeg-compression standard. It will also be compared to the classical DFT and the standard RDFS with $\alpha = 1.4$, which was prone to be an optimal value [3]. The vibration pattern that will be used is composed of 3 complex poles in both x and y-direction:

$$z(x_k, y_l) = e^{i0.1x_k}e^{i3.5y_l} + e^{i1.5x_k}e^{i2.3y_l} + e^{i3.7x_k}e^{i1.1y_l}, \tag{10}$$

where $x_k = 2\pi k/64$ and $y_l = 2\pi l/64$ with $k = 0, \ldots, 63$ and $l = 0, \ldots, 63$. 20 % noise was added on top of the vibration pattern. The vibration pattern with and without the added noise is shown in Fig. 1. The model order was increased sequentially for all 4 techniques, after which the mode shape error was calculated (Eq. (11)). For the ORDFS this means that $(2p+1) \times (2q+1)$ coefficients are estimated for the amplitudes $\mathbf{S}$. The model order was taken the same in both directions for simplicities sake, however, this is not a necessity. The same model orders or number of coefficients was taken for the other techniques as well.

$$\mathrm{err} = \frac{\sum_{n=0}^{N_k-1} |(z_{\mathrm{virtual}}(n) - z(n))|^2}{|z_{\mathrm{virtual}}(n)|^2}, \tag{11}$$

where z_{virtual} represents the data reduced mode shape.

In Table 1 the relative vibration pattern errors are listed for the DCT, DFT, RDFS and ORDFS techniques. It is clear that using the minimal order (in this case 49 coefficients; $p = q = 3$) the error is very high for the

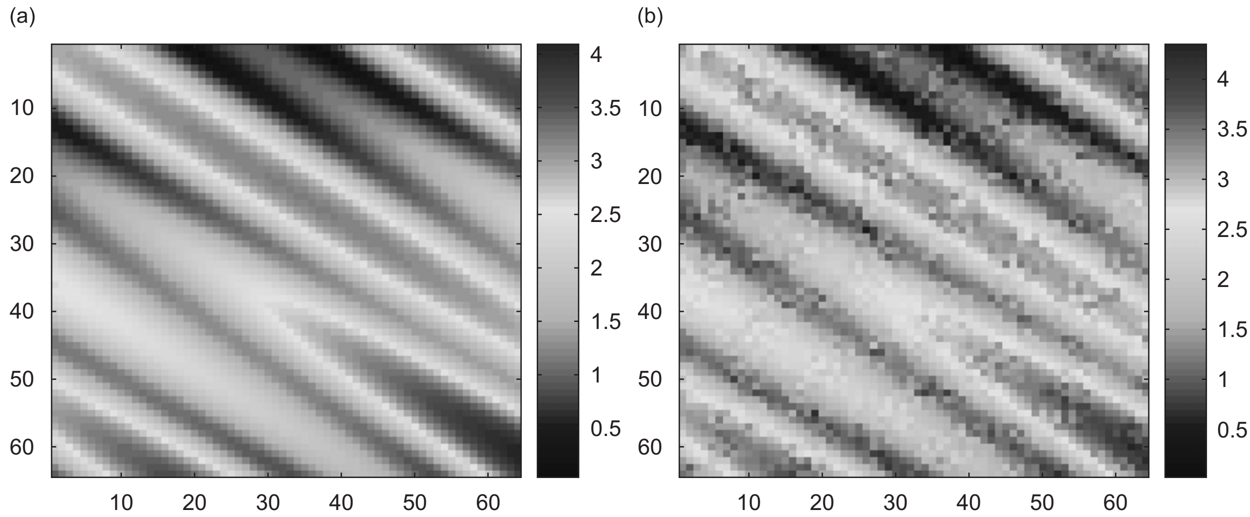

Fig. 1. Simulated vibration pattern $z(x_k, y_l)$. (a) noiseless and (b) with 20% Gaussian noise added.

Table 1
Relative vibration pattern error for the various simulated model orders for the DCT, DFT, RDFS and ORDFS technique

Error (%)	Order (3,3)	Order (4,4)	Order (5,5)
DCT	3.87	1.66	1.10
DFT	5.98	4.43	3.44
RDFS	56.36	9.4	0.85
ORDFS	61.87	0.47	0.14

ORDFS and RDFS, even higher than for both other techniques. However, as the model order increases the error value drops massively and the ORDFS approach proves superior .

In the next section, the proposed ORDFS method will be used to reduce data for measurements on a composite beam and a car door.

4. Experiments

4.1. 1D-experiments on an inorganic phosphate cement beam

In this section an experiment carried out with a Polytec PSV 300 laser vibrometer [10] on a inorganic phosphate cement (IPC) composite beam will be tackled using the ORDFS technique. Fig. 2 shows the test set-up.

The results will be compared with the same techniques mentioned in Section 3. The beam was excited acoustically with a swept sine and was suspended in a 'free-free' manner. For illustrative purposes we show the results of an operational deflection shape around the second mode (Fig. 3). Data reduction will again be applied directly to the deflection shape.

Starting from the estimated mode shape Fig. 3 illustrates that with a DFT analysis, using the seven most important DFT lines of the spectrum in (a), the synthesis is at best poor (12.07% relative error). The DCT analysis is much better but obviously still does not suffice (relative error 1.78%). Using the traditional RDFS method with $\alpha = 1.4$ again, the relative error is quite low at 0.40%. The best fit, however, is obtained by the ORDFS technique with a relative error of 0.01%. To achieve the same relative error between synthesis and measurement with the DFT analysis compared to the ORDFS with nine coefficients, no less than 168 DFT lines are necessary (180 lines would give an exact solution as this was the Nyquist frequency).

In Fig. 4 the relative error as a function of the compression ration is shown for the 4 different techniques that were used.

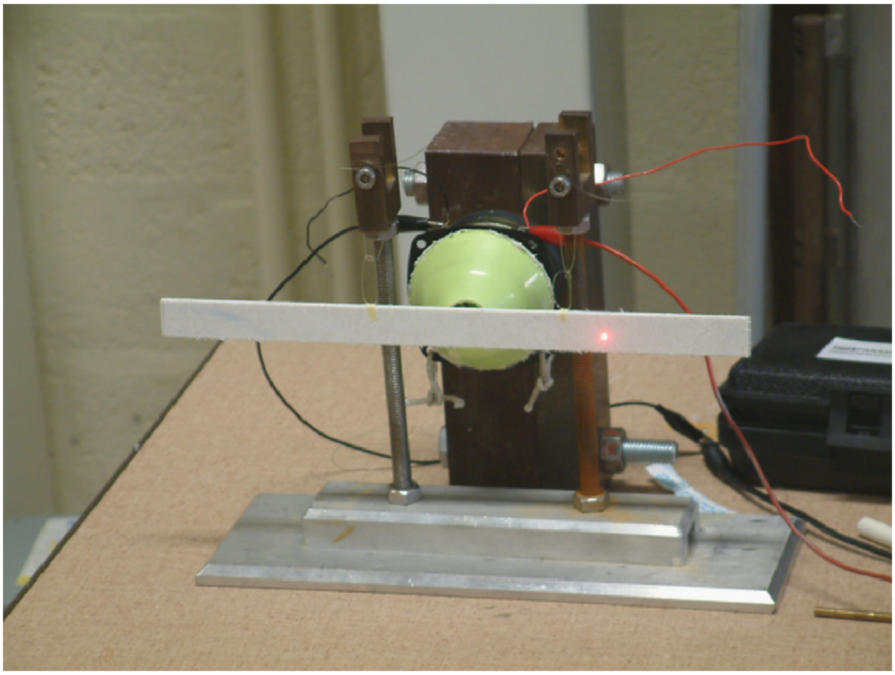

Fig. 2. Test set-up.

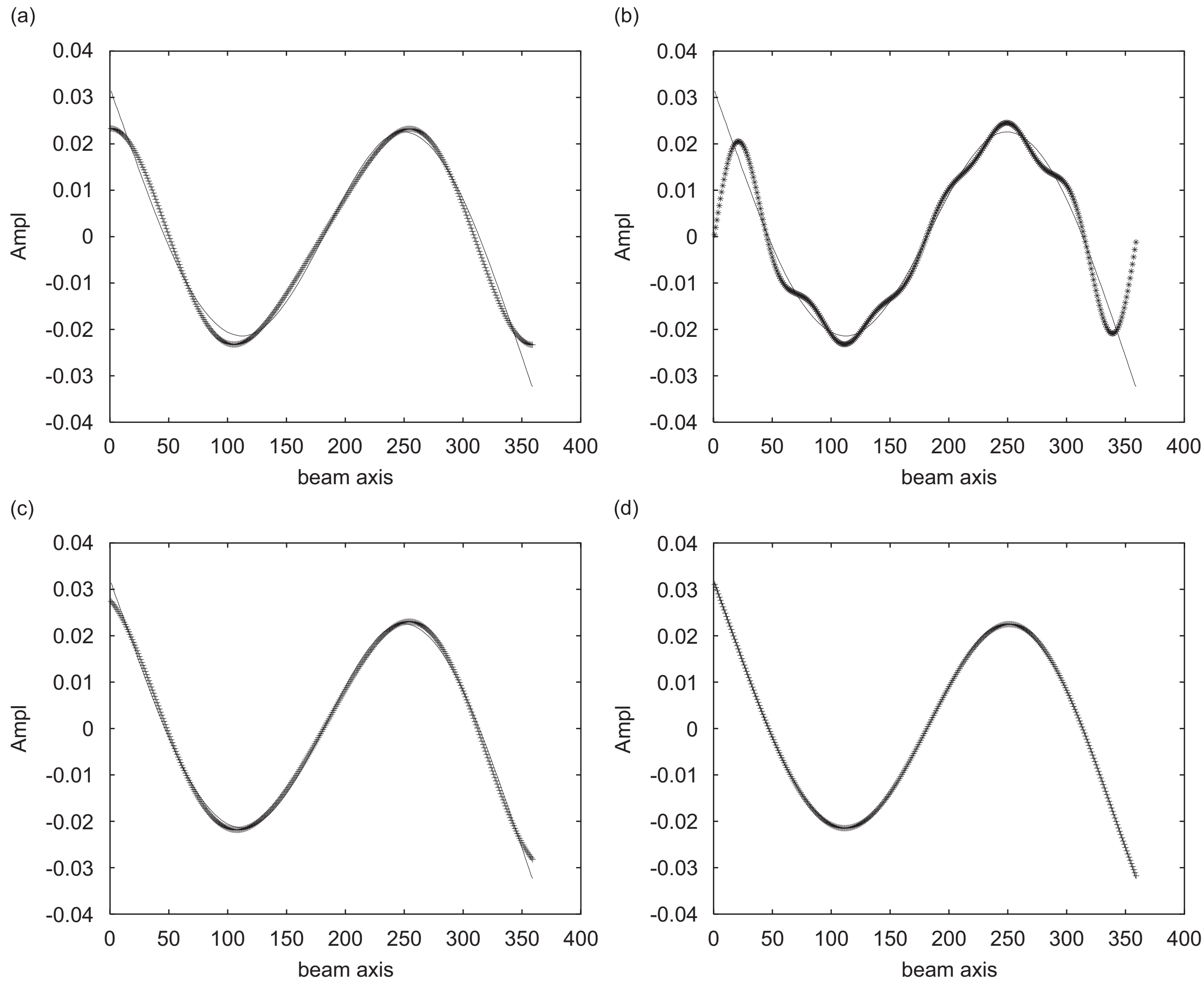

Fig. 3. Second mode shape of an IPC beam using 7 coefficients: (a) DCT analysis, (b) DFT analysis, (c) RDFS analysis, (d) ORDFS analysis and [solid curves indicate mode shapes and +-starred curves indicate synthesis.

4.2. SLDV measurements on a car door

In this section measurements are shown that were obtained from a car door (Fig. 5) using Polytec PSV 300 laser Doppler vibrometer [10]. The measurement grid had a rectangular shape (65×50 cm; 39×25 grid points) via a shaker with a swept sine excitation (0–200 Hz; 1 Hz resolution sampled at 512 Hz). The shaker was mounted in one of the door handle attachment points.

The mode shape that will be tackled is the second bending mode at 70 Hz. The measurements have a signal-to-noise ratio of 20 dB. This is quite low, but has been done deliberately, as to show the smoothing capabilities of the ORDFS technique.

Fig. 6 shows the mode shape under inspection and its frequency spectrum (only the real parts of the images are displayed; the imaginary parts give very similar results). Again data reduction will be applied directly to the estimated mode shape. Fig. 7 shows a comparison of the mode shapes estimated with the DCT, DFT, RDFS and ORDFS techniques using nine components. This value was chosen as it represents the maximum possible compression using the regressive techniques. For the classical RDFS technique α and β were both chosen equal to 1.4 [3]. In Fig. 8 the relative error of the four analysis methods from Fig. 7 is shown as

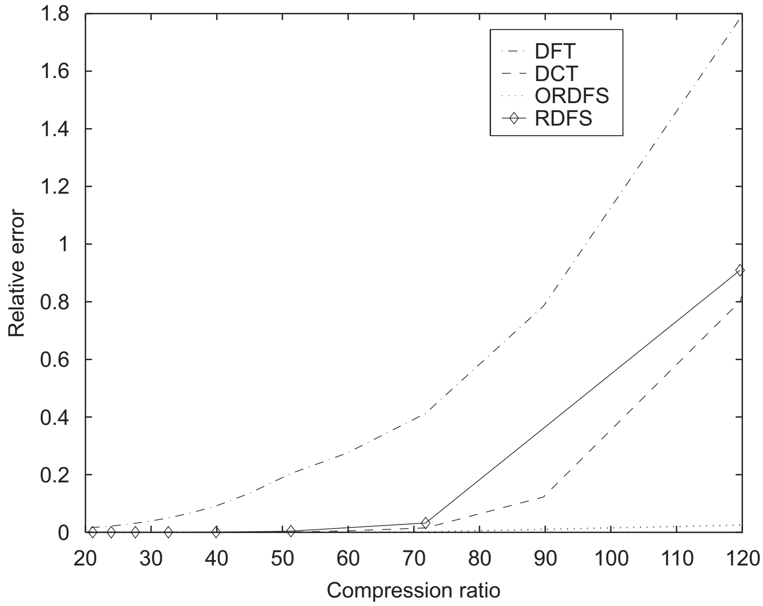

Fig. 4. Relative error as a function of the compression ratio for the DFT, DCT RDFS and ORDFS technique; (. -line indicates DFT, - - DCT, . . ORDFS and - ○ RDFS).

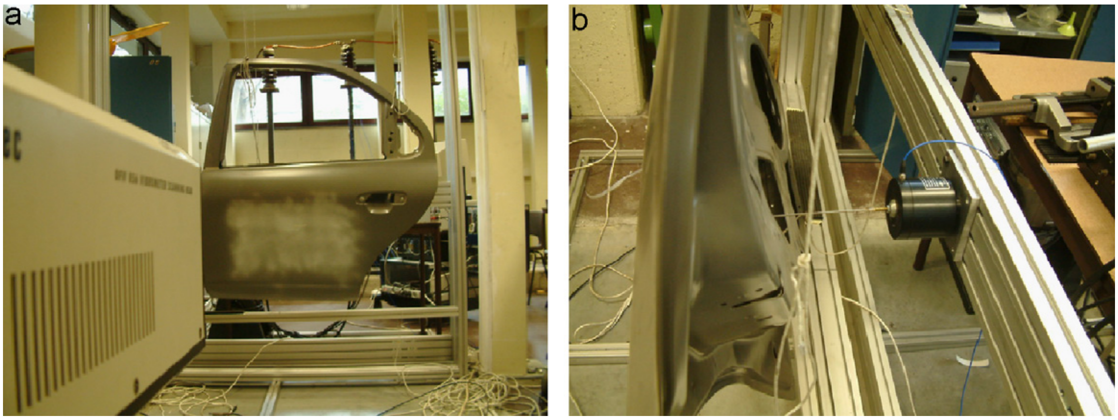

Fig. 5. Measurement set-up of a car door with a SLDV.

a function of the compression ratio. It is clear that for this two dimensional vibration pattern the same order is respected as for the one dimensional IPC second bending mode. From Fig. 8 it can be concluded that the classical RDFS performs worst for high compression ratios, despite the fact that the values chosen for α and β should be optimal [3]. At very low compression ratios it performs almost as well as the ORDFS technique, however, they are already quite low for the purpose at hand. The DFT does not perform exceptionally anywhere over the compression range. For high compression ratios the ORDFS performs best, while at very low compression ratios, the DCT technique proves superior. However, the relative error does not increase much with increasing compression ratios, which is not the case for both other techniques discussed here.

The importance of the correct estimation of the α and β coefficients in the ORDFS technique is shown in Fig. 9 where the relative error is shown as a function of α and β. By estimating the correct α and β for a given compression ratio and then varying them in turn, the relative error is calculated. It is clear that for this particular case (maximum compression) the correct estimation of these parameters is of great importance.

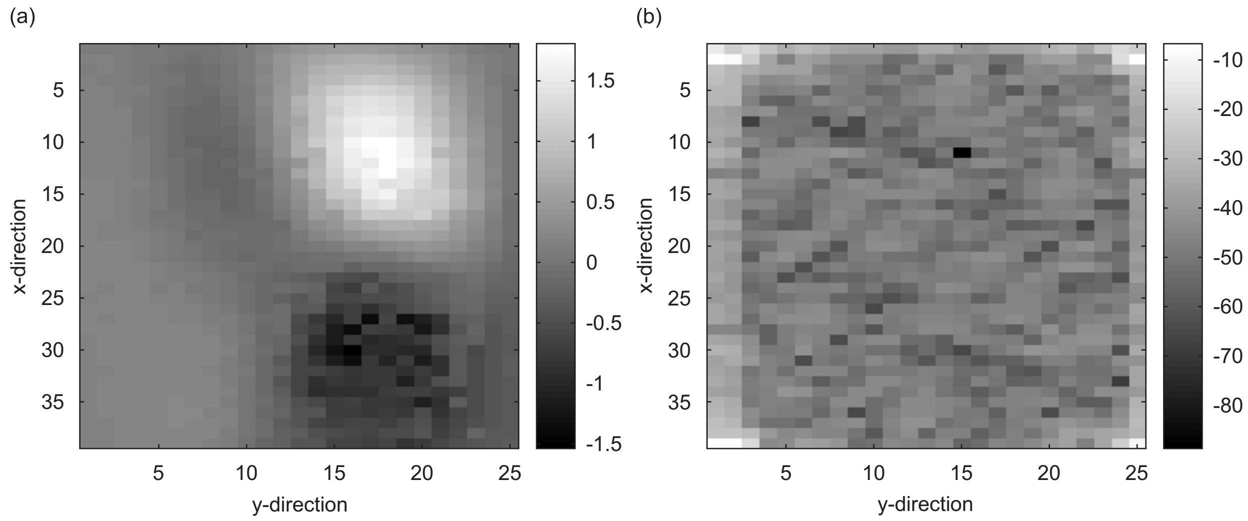

Fig. 6. (a) image of the second bending mode of a car door in mm/s. (b) frequency spectrum of image (a) in dB.

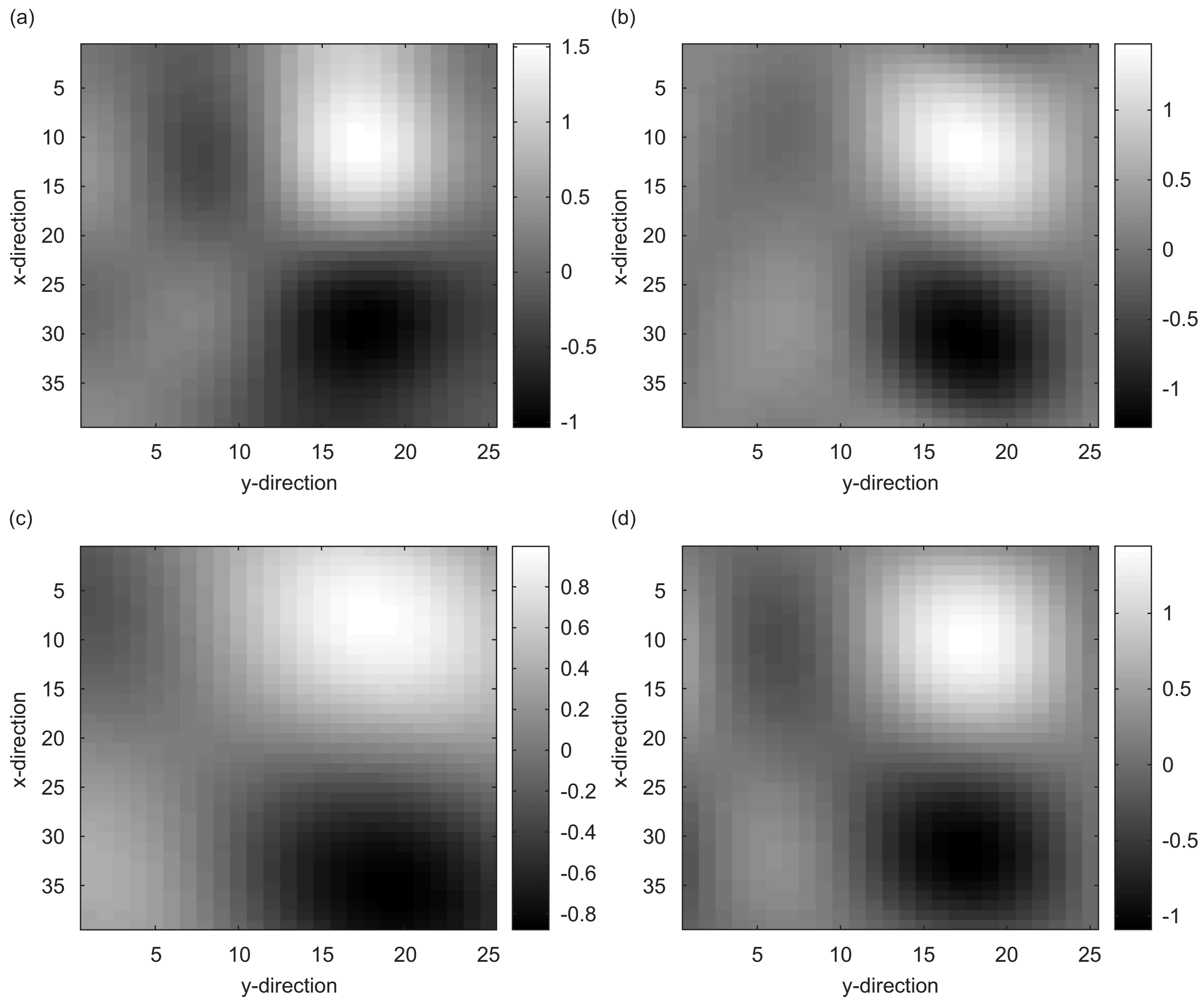

Fig. 7. Synthesis of Fig. 6(a) using 9 coefficients by means of: (a) a DCT analysis, (b) a DFT analysis, (c) a RDFS analysis, and (d) an ORDFS analysis (in mm/s).

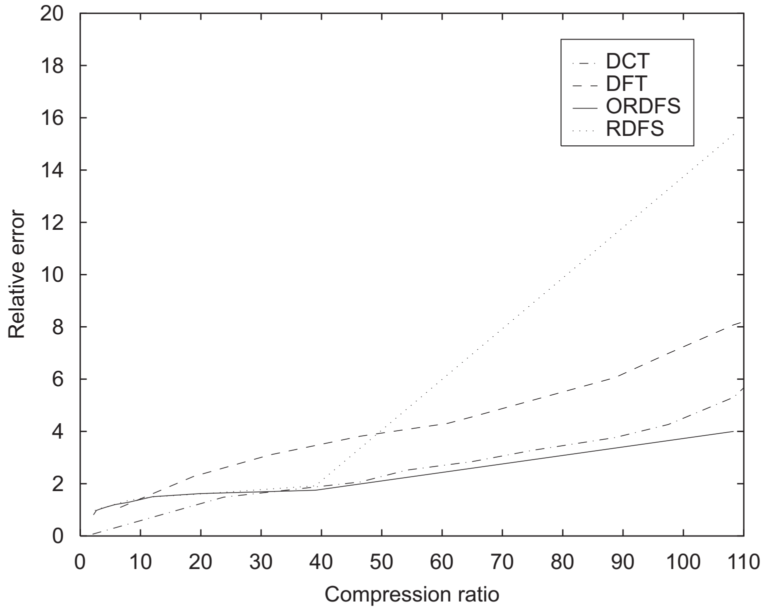

Fig. 8. Relative error (in %) as a function of the compression ratio for the DCT, DFT, RDFS and ORDFS compression methods (. -line indicates DFT, - - DCT, – ORDFS and . . RDFS).

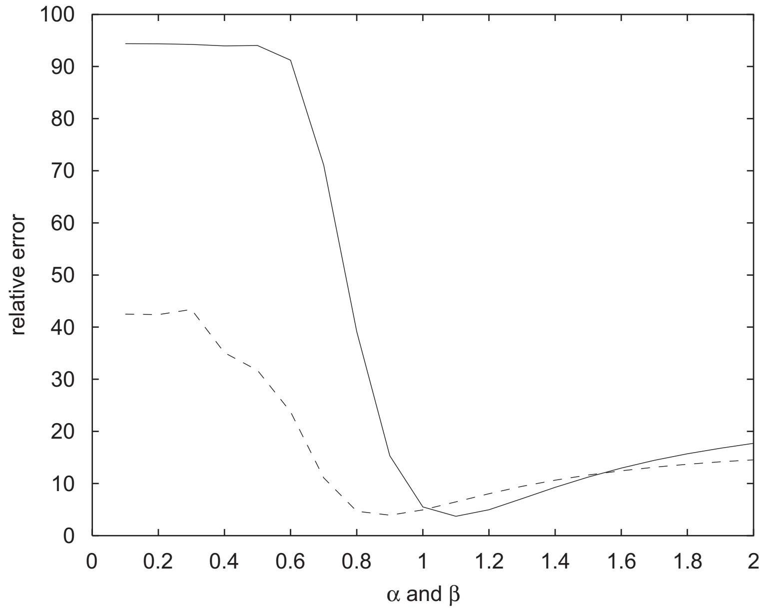

Fig. 9. Influence of α and β for a maximum compression ratio on the relative error of the analysis.

When decreasing the compression ratio, their contribution to a correct synthesis becomes less significant. These conclusions complement those that were deducted from the results shown in Fig. 8.

The calculation times for the different techniques for each experiment discussed in this section are shown in Table 2. The DCT, DFT and RDFS techniques exhibit roughly the same calculation time for each respective example. The ORDFS is computationally more complex due to the least squares problem which must be solved over a number of iterations.

Table 2
Calculation times for the DCT, DFT, RDFS, and ORDFS techniques on the IPC beam (9 coefficients used) and car door (9 coefficients used)

Calculation Time (s)	IPC beam	Car door (full)
DCT	0.08	0.07
DFT	0.01	0.08
RDFS	0.06	0.08
ORDFS	0.53	1.01

5. Conclusions

In this article a novel data reduction method based on the RDFS approach introduced by Arruda, was shown. This ORDFS was obtained by applying a nonlinear least-squares approach to estimate *a priori* unknown coefficients. The technique proved quite robust and attained higher compression ratios than other frequently used reduction techniques. The technique was compared to the existing DCT, DFT as well as the classical RDFS, and proved more accurate on almost all accounts for equal compression ratios. This was shown on simulations as well as real experiments on an IPC composite beam and a car door.

Acknowledgments

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References

[1] R. Crane, *A Simplified Approach to Image Processing, Classical and Modern Techniques in C*, Prentice Hall, New Jersey, 1997.
[2] M. Narasimha, A. Peterson, Computation of the discrete cosine transform, *IEEE Transactions on Communications* 26 (1978) 934–936.
[3] J. Arruda, Surface smoothing and partial derivatives computation using a regressive discrete fourier series, *Mechanical Systems and Signal Processing* 6 (1992) 41–50.
[4] J. Arruda, S.A.V. Rio, L.A.S.B. Santos, A space-frequency data compression method for spatially dense laser doppler vibrometer measurements, *Journal of Shock and Vibration* 3 (1996) 127–133.
[5] J. Vanherzeele, P. Guillaume, S. Vanlanduit, Improved Fourier analysis using parametric frequency-domain transfer-function estimators, *Mechanical Systems and Signal Processing* 21 (2007) 1704–1716.
[6] J. Vanherzeele, P. Guillaume, S. Vanlanduit, P. Verboven, Data reduction using a regressive discrete Fourier series, *Journal of Sound and Vibration* 298 (2006) 1–11.
[7] J. Vanherzeele, P. Guillaume, S. Vanlanduit, Data reduction using a regressive discrete Fourier series, in: 6th International conference on vibration measurements by laser techniques: advances and applications, E. Tomasini (Ed.), vol. 5503, SPIE Proceedings, 2004.
[8] J. Vanherzeele, Design of Regressive Fourier Series for Processing Optical Measurements, PhD thesis, Vrije Universiteit Brussel, 2007.
[9] Signal Processing Toolbox—For Use with MATLAB, The MathWorks, User's Guide.
[10] Polytec Scanning Vibrometer PSV 300 Hardware Manual.